Ellipsoids

- ellipsoids
- ellipsoids in estimation
Ellipsoids

if $A = A^T > 0$, the set

$$\mathcal{E} = \{ x | x^T A x \leq 1 \}$$

is an ellipsoid in $\mathbb{R}^n$, centered at 0
Ellipsoids

semi-axes are given by $s_i = \lambda_i^{-1/2} q_i$, i.e.:

- eigenvectors determine directions of semi-axes
- eigenvalues determine lengths of semi-axes

note:

- in direction $q_1$, $x^T A x$ is large, hence ellipsoid is thin in direction $q_1$
- in direction $q_n$, $x^T A x$ is small, hence ellipsoid is fat in direction $q_n$
- $\sqrt{\lambda_{\text{max}}/\lambda_{\text{min}}}$ gives maximum eccentricity

if $\mathcal{E} = \{ x \mid x^T B x \leq 1 \}$, where $B > 0$, then $\mathcal{E} \subseteq \hat{\mathcal{E}} \iff A \succeq B$