Overview

- course mechanics
- outline & topics
- what is a linear dynamical system?
- why study linear systems?
- some examples

Lecture notes and course materials originally by Stephen Boyd. Revisions by Sanjay Lall.
Course mechanics

- class web page: ee263.stanford.edu
Prerequisites

- exposure to linear algebra
- exposure to differential equations

*not needed*, but might increase appreciation:

- control systems
- circuits & systems
- dynamics
- probability theory
Major topics & outline

- linear algebra & applications
- autonomous linear dynamical systems
- linear dynamical systems with inputs & outputs
- basic quadratic control & estimation
Linear dynamical system

Continuous-time linear dynamical system (CT LDS) has the form

\[
\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)
\]

where:

- \( t \in \mathbb{R} \) denotes \textit{time}
- \( x(t) \in \mathbb{R}^n \) is the \textit{state} (vector)
- \( u(t) \in \mathbb{R}^m \) is the \textit{input} or \textit{control}
- \( y(t) \in \mathbb{R}^p \) is the \textit{output}
- \( A(t) \in \mathbb{R}^{n \times n} \) is the \textit{dynamics matrix}
- \( B(t) \in \mathbb{R}^{n \times m} \) is the \textit{input matrix}
- \( C(t) \in \mathbb{R}^{p \times n} \) is the \textit{output} or \textit{sensor matrix}
- \( D(t) \in \mathbb{R}^{p \times m} \) is the \textit{feedthrough matrix}
Some LDS terminology

\[ \dot{x} = Ax \]

- most linear systems encountered are \textit{time-invariant}: \( A, B, C, D \) are constant, \textit{i.e.}, don't depend on \( t \)

- when there is no input \( u \) (hence, no \( B \) or \( D \)) system is called \textit{autonomous}

- very often there is no feedthrough, \textit{i.e.}, \( D = 0 \)

- when \( u(t) \) and \( y(t) \) are scalar, system is called \textit{single-input, single-output} (SISO); when input & output signal dimensions are more than one, MIMO
Linear dynamical system

\[
\frac{dx}{dt} \quad \rightarrow \quad \dot{x} \\
\mathbf{x}(t) \quad \rightarrow \quad \mathbf{x}
\]

for lighter appearance, equations are often written

\[
\dot{x} = Ax + Bu, \quad y = Cx + Du
\]

- CT LDS is a first order vector \textit{differential equation}

- also called \textit{state equations}, or ‘\textit{m}-input, \textit{n}-state, \textit{p}-output’ LDS
Discrete-time linear dynamical system

A discrete-time linear dynamical system (DT LDS) has the form

\[ x(t + 1) = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t) \]

where

- \( t \in \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\} \)
- (vector) signals \( x, u, y \) are sequences

DT LDS is a first-order vector recursion
Why study linear systems?

applications arise in many areas, e.g.

- automatic control systems
- signal processing
- communications
- economics, finance
- circuit analysis, simulation, design
- mechanical and civil engineering
- aeronautics
- navigation, guidance
- machine learning
Origins and history

- parts of LDS theory can be traced to 19th century
- builds on classical circuits & systems (1920s on) (transfer functions ...) but with more emphasis on linear algebra
- first engineering application: aerospace, 1960s
- transitioned from specialized topic to ubiquitous in 1980s (just like digital signal processing, information theory, ... )
many dynamical systems are **nonlinear** (a fascinating topic) so why study **linear** systems?

- most techniques for nonlinear systems are based on linear methods
- methods for linear systems often work unreasonably well, in practice, for nonlinear systems
- if you don’t understand linear dynamical systems you certainly can’t understand nonlinear dynamical systems