

Ellipsoids

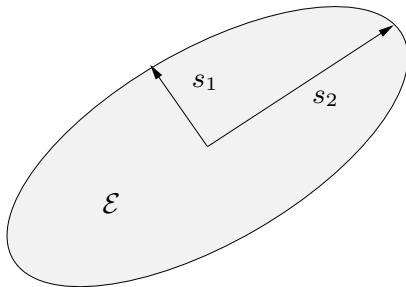
- ▶ ellipsoids
- ▶ ellipsoids in estimation

Ellipsoids

if $A = A^T > 0$, the set

$$\mathcal{E} = \{ x \mid x^T A x \leq 1 \}$$

is an *ellipsoid* in $\mathbb{R}^n$, centered at 0



Ellipsoids

semi-axes are given by $s_i = \lambda_i^{-1/2} q_i$, i.e.:

- ▶ eigenvectors determine directions of semiaxes
- ▶ eigenvalues determine lengths of semiaxes

note:

- ▶ in direction q_1 , $x^T A x$ is *large*, hence ellipsoid is *thin* in direction q_1
- ▶ in direction q_n , $x^T A x$ is *small*, hence ellipsoid is *fat* in direction q_n
- ▶ $\sqrt{\lambda_{\max}/\lambda_{\min}}$ gives maximum *eccentricity*

if $\tilde{\mathcal{E}} = \{ x \mid x^T B x \leq 1 \}$, where $B > 0$, then $\mathcal{E} \subseteq \tilde{\mathcal{E}} \iff A \geq B$