

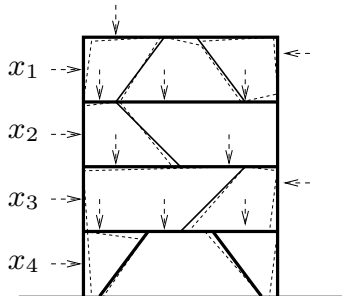
Engineering examples

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Linear elastic structure

- ▶ x_j is external force applied at some node, in some fixed direction
- ▶ y_i is (small) deflection of some node, in some fixed direction



(provided x , y are small) we have $y \approx Ax$

- ▶ A is called the *compliance matrix*
- ▶ a_{ij} gives deflection i per unit force at j (in m/N)

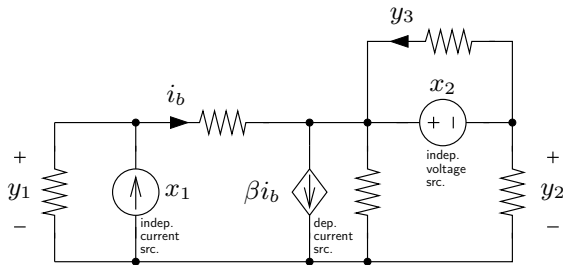
Linear elastic structure – Questions

Suppose $y_1, \dots, y_4$ represent the displacements at the four nodes on the right side of the object, with the positive direction taken to be to the right.

- ▶ What can we say about the sign of a_{ij} ?
- ▶ How do the entries in the first row of A compare?
- ▶ How do the entries in the first column of A compare?
- ▶ What is the largest entry of A ?
- ▶ What is the smallest entry of A ?

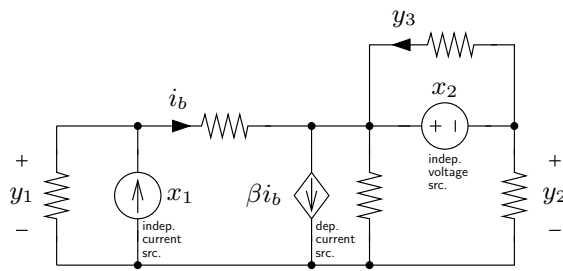
Linear static circuit

Interconnection of resistors, linear dependent (controlled) sources, and independent sources



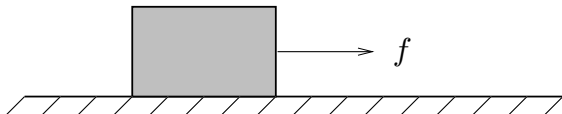
- ▶ x_j is value of independent source j
- ▶ y_i is some circuit variable (voltage, current)
- ▶ we have $y = Ax$
- ▶ if x_j are currents and y_i are voltages, A is called the *impedance* or *resistance* matrix

Linear static circuit – Questions



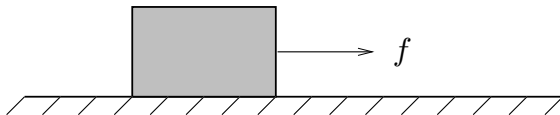
- What is the value of a_{32} ?
- What is the value of a_{31} ?
- What is the sign of a_{11} ?

Final position/velocity of mass due to applied forces



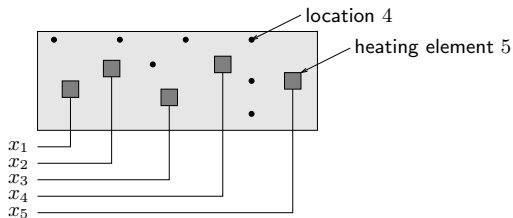
- ▶ unit mass, zero position/velocity at $t = 0$, subject to force $f(t)$ for $0 \leq t \leq n$
- ▶ $f(t) = x_j$ for $j - 1 \leq t < j$, $j = 1, \dots, n$
(x is the sequence of applied forces, constant in each interval)
- ▶ y_1, y_2 are final position and velocity (*i.e.*, at $t = n$)
- ▶ we have $y = Ax$
- ▶ a_{1j} gives influence of applied force during $j - 1 \leq t < j$ on final position
- ▶ a_{2j} gives influence of applied force during $j - 1 \leq t < j$ on final velocity

Final position/velocity of mass due to applied forces – Questions



- ▶ How do the entries in the second row of A compare?
- ▶ How do the entries in the first row of A compare?
- ▶ What is the value of a_{21} ?
- ▶ What is the value of a_{1n} ?
- ▶ What is the value of a_{2n} ?
- ▶ If we have a force constraint of the form $|x_1| + |x_2| + \cdots + |x_n| = 1$, what is the farthest we can move the object in 10 seconds?
- ▶ How can we move the object 1 meter in 4 seconds so that it comes to rest at that point? Is the solution unique?

Thermal system



- ▶ x_j is power of j th heating element or heat source
- ▶ y_i is change in steady-state temperature at location i
- ▶ thermal transport via conduction
- ▶ $y = Ax$
- ▶ a_{ij} gives influence of heater j at location i (in $^{\circ}\text{C}/\text{W}$)
- ▶ j th column of A gives pattern of steady-state temperature rise due to 1W at heater j
- ▶ i th row shows how heaters affect location i