Ellipsoids

- ellipsoids
- ellipsoids in estimation

Ellipsoids

if $A = A^{\mathsf{T}} > 0$, the set

$$\mathcal{E} = \{\, x \mid x^{\mathsf{T}} A x \le 1 \,\}$$

is an *ellipsoid* in $\mathbb{R}^n$, centered at 0

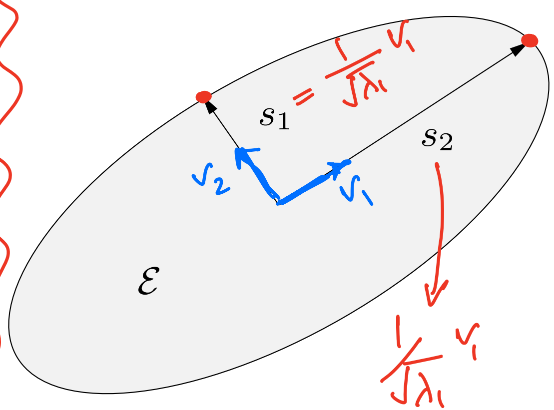
$x = \alpha v$: eigenvector $(\|v\| = 1)$

$$x^T A x = \alpha^2 \cdot v^T \underbrace{A v}_{\lambda v}$$

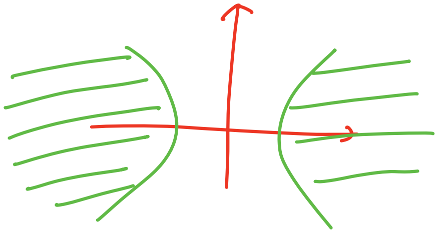
$$= \alpha^2 \lambda \underbrace{v^T v}_{\|v\|=1} = \boxed{\alpha^2 \lambda}$$

Boundary : $x^T A x = 1$

$$\alpha \lambda^2 = 1 \to \alpha = \frac{1}{\sqrt{\lambda}}$$



$\lambda_i < 0$



$A \lesssim 0$

$$\mathcal{E} = \{x \mid x^T A x \le 1\} = \mathbb{R}^n$$

Ellipsoids

$\underline{\mathbb{R}^3}$:

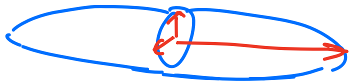
λ_1, λ_2 large

λ_3 small

semi-axes are given by $s_i = \lambda_i^{-1/2} q_i$, *i.e.*:

- eigenvectors determine directions of semiaxes
- eigenvalues determine lengths of semiaxes

cigar



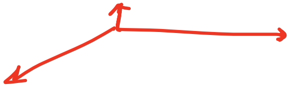
note:

- in direction q_1, $x^\mathsf{T} A x$ is *large*, hence ellipsoid is *thin* in direction q_1
- in direction q_n, $x^\mathsf{T} A x$ is *small*, hence ellipsoid is *fat* in direction q_n
- $\sqrt{\lambda_{\max}/\lambda_{\min}}$ gives maximum *eccentricity*

λ_1 large

λ_2, λ_3 small

pancake



if $\tilde{\mathcal{E}} = \{ x \mid x^\mathsf{T} B x \le 1 \}$, where $B > 0$, then $\mathcal{E} \subseteq \tilde{\mathcal{E}} \iff A \ge B$