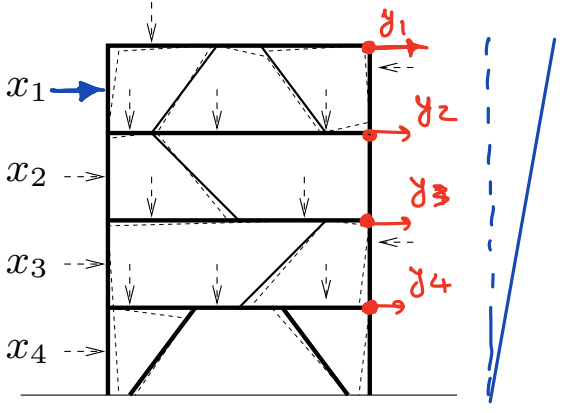
Engineering examples

Stephen Boyd and Sanjay Lall
Revision by Amirhossein Afsharrad

EE263
Stanford University

Linear elastic structure

- x_j is external force applied at some node, in some fixed direction
- y_i is (small) deflection of some node, in some fixed direction



a_{ij}
output
input

(provided x, y are small) we have $y \approx Ax$

- A is called the *compliance matrix*
- a_{ij} gives deflection i per unit force at j (in m/N)

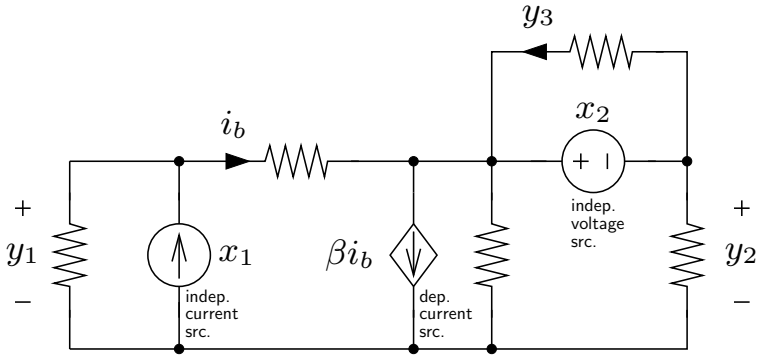
Linear elastic structure – Questions

Suppose $y_1, \ldots, y_4$ represent the displacements at the four nodes on the right side of the object, with the positive direction taken to be to the right.

- What can we say about the sign of a_{ij}? $a_{ij} > 0$
- How do the entries in the first row of A compare? $a_{11} > a_{12} > a_{13} > a_{14}$
- How do the entries in the first column of A compare? $a_{11} > a_{21} > a_{31} > a_{41}$
- What is the largest entry of A? a_{11}
- What is the smallest entry of A? probably a_{41}

Linear static circuit

Interconnection of resistors, linear dependent (controlled) sources, and independent sources

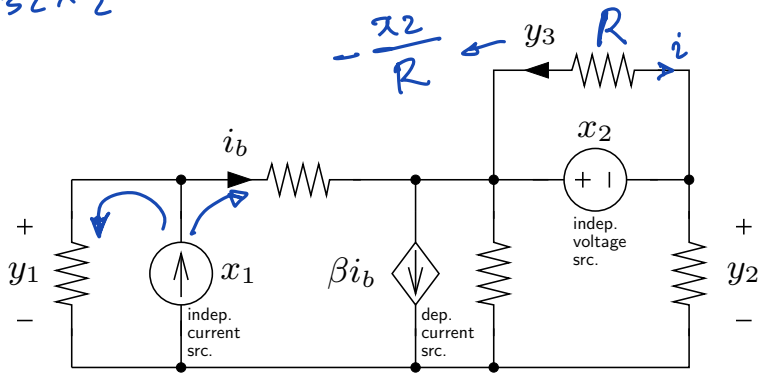


- x_j is value of independent source j
- y_i is some circuit variable (voltage, current)
- we have $y = Ax$
- if x_j are currents and y_i are voltages, A is called the *impedance* or *resistance* matrix

Linear static circuit – Questions

$y_3 = a_{31} x_1 + a_{32} x_2$

$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



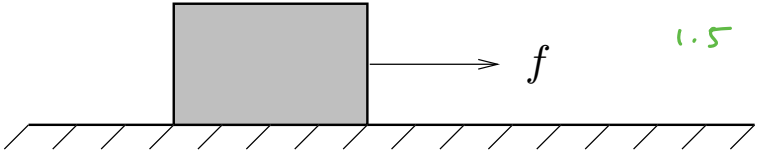
$-\frac{x_2}{R}$

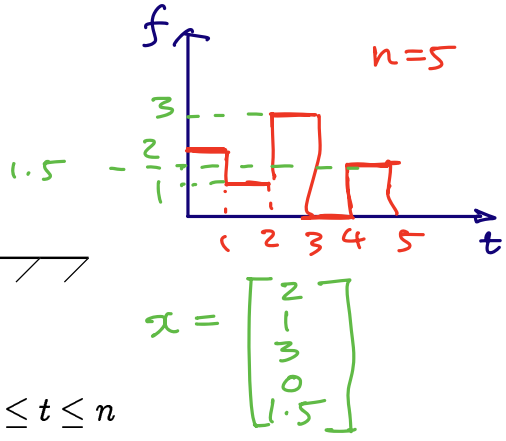
- What is the value of a_{32}? $\frac{-1}{R}$
- What is the value of a_{31}? 0
- What is the sign of a_{11}? $+$

$y_1 = a_{11} x_1 + a_{12} x_2$

$x_1 = 1$, $x_2 = 0$

Final position/velocity of mass due to applied forces



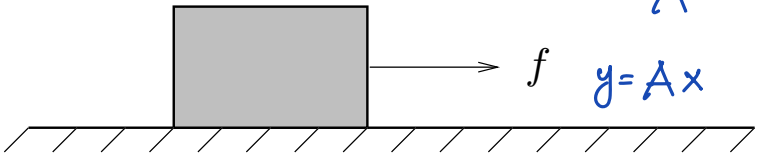


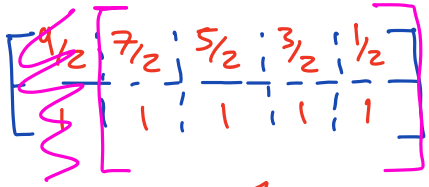
- unit mass, zero position/velocity at $t = 0$, subject to force $f(t)$ for $0 \leq t \leq n$
- $f(t) = x_j$ for $j - 1 \leq t < j$, $j = 1, \ldots, n$
  (x is the sequence of applied forces, constant in each interval)
- y_1, y_2 are final position and velocity ($i.e.$, at $t = n$)
- we have $y = Ax$
- a_{1j} gives influence of applied force during $j - 1 \leq t < j$ on final position
- a_{2j} gives influence of applied force during $j - 1 \leq t < j$ on final velocity

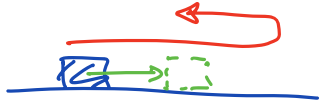
Final position/velocity of mass due to applied forces – Questions

$f = m \cdot a$

$f = a = \frac{\Delta v}{\Delta t}$, $\Delta t = 1$



$A =$ 



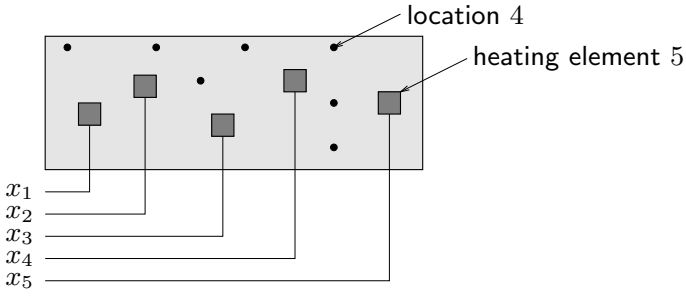
- How do the entries in the second row of A compare? $a_{21} = a_{22} = a_{23} = a_{24} = a_{25}$
- How do the entries in the first row of A compare? $a_{11} > a_{12} > \cdots > a_{15}$
- What is the value of a_{21}?
- What is the value of a_{1n}?
- What is the value of a_{2n}?

$x = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{cases} 1 = \frac{7}{2}x_1 + \frac{5}{2}x_2 + \frac{3}{2}x_3 + \frac{1}{2}x_4 \\ 0 = x_1 + x_2 + x_3 + x_4 \end{cases}$

- If we have a force constraint of the form $|x_1| + |x_2| + \cdots + |x_n| = 1$, what is the farthest we can move the object in 10 seconds?
- How can we move the object 1 meter in 4 seconds so that it comes to rest at that point? Is the solution unique?

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} & A & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

Thermal system



- x_j is power of jth heating element or heat source
- y_i is change in steady-state temperature at location i
- thermal transport via conduction
- $y = Ax$
- a_{ij} gives influence of heater j at location i (in °C/W)
- jth column of A gives pattern of steady-state temperature rise due to 1W at heater j
- ith row shows how heaters affect location i