Lecture 1
Matrix Terminology and Notation

- matrix dimensions
- column and row vectors
- special matrices and vectors
Matrix dimensions

A matrix is a rectangular array of numbers between brackets.

Examples:

\[
A = \begin{bmatrix}
0 & 1 & -2.3 & 0.1 \\
1.3 & 4 & -0.1 & 0 \\
4.1 & -1 & 0 & 1.7
\end{bmatrix}, \quad
B = \begin{bmatrix}
3 & -3 \\
12 & 0
\end{bmatrix}
\]

dimension (or size) always given as (numbers of) rows × columns.

- A is a 3 × 4 matrix, B is 2 × 2.
- The matrix A has four columns; B has two rows.

An m × n matrix is called square if m = n, fat if m < n, skinny if m > n.
Matrix coefficients

**coefficients** (or entries) of a matrix are the values in the array

coefficients are referred to using double subscripts for row, column

$A_{ij}$ is the value in the $i$th row, $j$ column of $A$; also called $i, j$ entry of $A$

$i$ is the **row index** of $A_{ij}$; $j$ is the **column index** of $A_{ij}$

(here, $A$ is a matrix; $A_{ij}$ is a number)

**example:** for $A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}$, we have:

$A_{23} = -0.1$, $A_{22} = 4$, but $A_{41}$ is meaningless

the row index of the entry with value $-2.3$ is $1$; its column index is $3$
Column and row vectors

A matrix with one column, \( i.e., \) size \( n \times 1 \), is called a (column) **vector**

A matrix with one row, \( i.e., \) size \( 1 \times n \), is called a **row vector**

‘vector’ alone usually refers to column vector

We give only one index for column & row vectors and call entries **components**

\[
v = \begin{bmatrix} 1 \\ -2 \\ 3.3 \\ 0.3 \end{bmatrix} \quad w = \begin{bmatrix} -2.1 & -3 & 0 \end{bmatrix}
\]

- \( v \) is a 4-vector (or \( 4 \times 1 \) matrix); its third component is \( v_3 = 3.3 \)
- \( w \) is a row vector (or \( 1 \times 3 \) matrix); its third component is \( w_3 = 0 \)
Matrix equality

\[ A = B \] means:

- \( A \) and \( B \) have the same size
- the corresponding entries are equal

for example,

- \[
\begin{bmatrix}
-2 \\
3.3
\end{bmatrix}
\neq
\begin{bmatrix}
-2 & -3.3
\end{bmatrix}
\] since the dimensions don’t agree
- \[
\begin{bmatrix}
-2 \\
3.3
\end{bmatrix}
\neq
\begin{bmatrix}
-2 \\
3.1
\end{bmatrix}
\] since the 2nd components don’t agree
Zero and identity matrices

$0_{m \times n}$ denotes the $m \times n$ zero matrix, with all entries zero.

$I_n$ denotes the $n \times n$ identity matrix, with

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$0_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, \hspace{1cm} $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$0_{n \times 1}$ called zero vector; $0_{1 \times n}$ called zero row vector

**convention:** usually the subscripts are dropped, so you have to figure out the size of $0$ or $I$ from context.
Unit vectors

e_i denotes the \(i\)th unit vector: its \(i\)th component is one, all others zero

the three unit 3-vectors are:

\[
e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

as usual, you have to figure the size out from context

unit vectors are the columns of the identity matrix \(I\)

some authors use \(1\) (or \(e\)) to denote a vector with all entries one, sometimes called the ones vector

the ones vector of dimension 2 is \(1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\)