

# Lecture 1

## Matrix Terminology and Notation

- matrix dimensions
- column and row vectors
- special matrices and vectors

# Matrix dimensions

a *matrix* is a rectangular array of numbers between brackets

**examples:**

$$A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -3 \\ 12 & 0 \end{bmatrix}$$

**dimension** (or size) always given as (numbers of) rows  $\times$  columns

- $A$  is a  $3 \times 4$  matrix,  $B$  is  $2 \times 2$
- the matrix  $A$  has four columns;  $B$  has two rows

$m \times n$  matrix is called *square* if  $m = n$ , *fat* if  $m < n$ , *skinny* if  $m > n$

# Matrix coefficients

**coefficients** (or entries) of a matrix are the values in the array

coefficients are referred to using double subscripts for row, column

$A_{ij}$  is the value in the  $i$ th row,  $j$  column of  $A$ ; also called  $i, j$  entry of  $A$

$i$  is the *row index* of  $A_{ij}$ ;  $j$  is the *column index* of  $A_{ij}$

(here,  $A$  is a matrix;  $A_{ij}$  is a number)

**example:** for  $A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}$ , we have:

$A_{23} = -0.1$ ,  $A_{22} = 4$ , but  $A_{41}$  is meaningless

the row index of the entry with value  $-2.3$  is 1; its column index is 3

## Column and row vectors

a matrix with one column, *i.e.*, size  $n \times 1$ , is called a (column) *vector*

a matrix with one row, *i.e.*, size  $1 \times n$ , is called a *row vector*

‘vector’ alone usually refers to column vector

we give only one index for column & row vectors and call entries *components*

$$v = \begin{bmatrix} 1 \\ -2 \\ 3.3 \\ 0.3 \end{bmatrix} \quad w = [ -2.1 \quad -3 \quad 0 ]$$

- $v$  is a 4-vector (or  $4 \times 1$  matrix); its third component is  $v_3 = 3.3$
- $w$  is a row vector (or  $1 \times 3$  matrix); its third component is  $w_3 = 0$

# Matrix equality

$A = B$  means:

- $A$  and  $B$  have the same size
- the corresponding entries are equal

for example,

- $\begin{bmatrix} -2 \\ 3.3 \end{bmatrix} \neq \begin{bmatrix} -2 & -3.3 \end{bmatrix}$  since the dimensions don't agree
- $\begin{bmatrix} -2 \\ 3.3 \end{bmatrix} \neq \begin{bmatrix} -2 \\ 3.1 \end{bmatrix}$  since the 2nd components don't agree

# Zero and identity matrices

$0_{m \times n}$  denotes the  $m \times n$  **zero matrix**, with all entries zero

$I_n$  denotes the  $n \times n$  **identity matrix**, with

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$0_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$0_{n \times 1}$  called *zero vector*;  $0_{1 \times n}$  called *zero row vector*

**convention:** usually the subscripts are dropped, so you have to figure out the size of  $0$  or  $I$  from context

# Unit vectors

$e_i$  denotes the  $i$ th **unit vector**: its  $i$ th component is one, all others zero

the three unit 3-vectors are:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

as usual, you have to figure the size out from context

unit vectors are the columns of the identity matrix  $I$

some authors use  $\mathbf{1}$  (or  $e$ ) to denote a vector with all entries one, sometimes called the **ones vector**

the ones vector of dimension 2 is  $\mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$