Overview

- course mechanics
- outline & topics
- what is a linear dynamical system?
- why study linear systems?
- some examples

Lecture notes and course materials originally by Stephen Boyd. Revisions by Sanjay Lall.

Course mechanics

- class web page: ee263.stanford.edu

Prerequisites

- exposure to linear algebra
- exposure to differential equations

not needed, but might increase appreciation:

- control systems
- circuits & systems
- dynamics

Major topics & outline

- linear algebra & applications
- autonomous linear dynamical systems
- linear dynamical systems with inputs & outputs
- basic quadratic control & estimation

Linear dynamical system

continuous-time linear dynamical system (CT LDS) has the form

$$\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)$$

where:

- $t \in \mathbb{R}$ denotes *time*
- $x(t) \in \mathbb{R}^n$ is the *state* (vector)
- $u(t) \in \mathbb{R}^m$ is the *input* or *control*
- $y(t) \in \mathbb{R}^p$ is the *output*
- $A(t) \in \mathbb{R}^{n \times n}$ is the *dynamics matrix*
- $B(t) \in \mathbb{R}^{n \times m}$ is the *input matrix*
- $C(t) \in \mathbb{R}^{p \times n}$ is the *output* or *sensor matrix*
- $D(t) \in \mathbb{R}^{p \times m}$ is the *feedthrough matrix*

Some LDS terminology

- most linear systems encountered are *time-invariant*: A, B, C, D are constant, $i.e.$, don't depend on t
- when there is no input u (hence, no B or D) system is called *autonomous*
- very often there is no feedthrough, $i.e.$, $D = 0$
- when $u(t)$ and $y(t)$ are scalar, system is called *single-input, single-output* (SISO); when input & output signal dimensions are more than one, MIMO

Linear dynamical system

for lighter appearance, equations are often written

$$\dot{x} = Ax + Bu, \qquad y = Cx + Du$$

- CT LDS is a first order vector *differential equation*
- also called *state equations*, or 'm-input, n-state, p-output' LDS

Discrete-time linear dynamical system

discrete-time linear dynamical system (DT LDS) has the form

$$x(t+1) = A(t)x(t) + B(t)u(t), \qquad y(t) = C(t)x(t) + D(t)u(t)$$

where

- $t \in \mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$
- (vector) signals x, u, y are *sequences*

DT LDS is a first-order vector *recursion*

Why study linear systems?

applications arise in **many** areas, $e.g.$

- automatic control systems
- signal processing
- communications
- economics, finance
- circuit analysis, simulation, design
- mechanical and civil engineering
- aeronautics
- navigation, guidance
- machine learning

Origins and history

- parts of LDS theory can be traced to 19th century
- builds on classical circuits & systems (1920s on) (transfer functions ...) but with more emphasis on linear algebra
- first engineering application: aerospace, 1960s
- transitioned from specialized topic to ubiquitous in 1980s
  (just like digital signal processing, information theory, ...)

Nonlinear dynamical systems

many dynamical systems are **nonlinear** (a fascinating topic) so why study **linear** systems?

- most techniques for nonlinear systems are based on linear methods
- methods for linear systems often work unreasonably well, in practice, for nonlinear systems
- if you don't understand linear dynamical systems you certainly can't understand nonlinear dynamical systems