Overview

- course mechanics
- outline & topics
- what is a linear dynamical system?
- why study linear systems?
- some examples

Lecture notes and course materials originally by Stephen Boyd. Revisions by Sanjay Lall.
Course mechanics

► all class info, lectures, homeworks on class web page:

      ee263.stanford.edu

► announcements and forum: piazza.com

► grades: canvas.stanford.edu

► mailing list: ee263-fall1617-staff@lists.stanford.edu

► lecture videos online: mvideox.stanford.edu
Course requirements

- weekly homework, due Friday at 4pm
- take-home midterm exam (10/29 – 10/30)
- take-home final exam (12/10 – 12/11)
- homework 30%, midterm 30%, final 40%

check the website for updates
Prerequisites

- exposure to linear algebra \((e.g.,\) Math 104)\)
- exposure to Laplace transform, differential equations
- exposure to probability and statistics

**not needed, but might increase appreciation:**

- control systems
- circuits & systems
- dynamics
Major topics & outline

- linear algebra & applications
- autonomous linear dynamical systems
- linear dynamical systems with inputs & outputs
- basic quadratic control & estimation
Linear dynamical system

*continuous-time* linear dynamical system (CT LDS) has the form

\[
\frac{dx}{dt} = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)
\]

where:

- \( t \in \mathbb{R} \) denotes *time*
- \( x(t) \in \mathbb{R}^n \) is the *state* (vector)
- \( u(t) \in \mathbb{R}^m \) is the *input* or *control*
- \( y(t) \in \mathbb{R}^p \) is the *output*
- \( A(t) \in \mathbb{R}^{n\times n} \) is the *dynamics matrix*
- \( B(t) \in \mathbb{R}^{n\times m} \) is the *input matrix*
- \( C(t) \in \mathbb{R}^{p\times n} \) is the *output* or *sensor matrix*
- \( D(t) \in \mathbb{R}^{p\times m} \) is the *feedthrough matrix*
Linear dynamical system

for lighter appearance, equations are often written

\[ \dot{x} = Ax + Bu, \quad y = Cx + Du \]

- CT LDS is a first order vector differential equation
- also called state equations, or ‘m-input, n-state, p-output’ LDS
Some LDS terminology

- most linear systems encountered are *time-invariant*: $A$, $B$, $C$, $D$ are constant, i.e., don’t depend on $t$

- when there is no input $u$ (hence, no $B$ or $D$) system is called *autonomous*

- very often there is no feedthrough, i.e., $D = 0$

- when $u(t)$ and $y(t)$ are scalar, system is called *single-input, single-output* (SISO); when input & output signal dimensions are more than one, MIMO
Discrete-time linear dynamical system

*discrete-time* linear dynamical system (DT LDS) has the form

\[
x(t + 1) = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t)
\]

where

- \( t \in \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\} \)
- (vector) signals \( x, u, y \) are *sequences*

DT LDS is a first-order vector *recursion*
Why study linear systems?

applications arise in many areas, e.g.

- automatic control systems
- signal processing
- communications
- economics, finance
- circuit analysis, simulation, design
- mechanical and civil engineering
- aeronautics
- navigation, guidance
Usefulness of LDS

- depends on availability of **computing power**, which is large & increasing exponentially

- used for
  - analysis & design
  - implementation, embedded in real-time systems
Origins and history

- parts of LDS theory can be traced to 19th century
- builds on classical circuits & systems (1920s on) (transfer functions . . . ) but with more emphasis on linear algebra
- first engineering application: aerospace, 1960s
- transitioned from specialized topic to ubiquitous in 1980s (just like digital signal processing, information theory, . . .)
Nonlinear dynamical systems

many dynamical systems are **nonlinear** (a fascinating topic) so why study **linear** systems?

- most techniques for nonlinear systems are based on linear methods
- methods for linear systems often work unreasonably well, in practice, for nonlinear systems
- if you don’t understand linear dynamical systems you certainly can’t understand nonlinear dynamical systems