Example: Least-squares navigation
Navigation from range measurements

navigation using range measurements from distant beacons

beacons far from unknown position $x \in \mathbb{R}^2$, so linearization around $x = 0$ (say) nearly exact
Navigation

ranges $y \in \mathbb{R}^4$ measured, with measurement noise $v$:

$$y = \begin{bmatrix} k_1^T \\ k_2^T \\ k_3^T \\ k_4^T \end{bmatrix} x + v$$

where $k_i$ is unit vector from 0 to beacon $i$

▶ problem: estimate $x \in \mathbb{R}^2$, given $y \in \mathbb{R}^4$

▶ measurement errors are independent, Gaussian, with standard deviation 2 (details not important, roughly a 2:1 measurement redundancy ratio)

▶ actual position is $x = (5.59, 10.58)$;

▶ measurement is $y = (-11.95, -2.84, -9.81, 2.81)$
Just enough measurements method

$y_1$ and $y_2$ suffice to find $x$ (when $v = 0$)

compute estimate $\hat{x}$ by inverting top $(2 \times 2)$ half of $A$:

$$\hat{x} = B_{je}y = \begin{bmatrix} 0 & -1.0 & 0 & 0 \\ -1.12 & 0.5 & 0 & 0 \end{bmatrix} y = \begin{bmatrix} 2.84 \\ 11.9 \end{bmatrix}$$

(norm of error: 3.07)
Least-squares method

compute estimate $\hat{x}$ by least-squares:

$$\hat{x} = A^\dagger y = \begin{bmatrix} -0.23 & -0.48 & 0.04 & 0.44 \\ -0.47 & -0.02 & -0.51 & -0.18 \end{bmatrix} y = \begin{bmatrix} 4.95 \\ 10.26 \end{bmatrix}$$

(norm of error: 0.72)

- $B_{je}$ and $A^\dagger$ are both left inverses of $A$
- larger entries in $B$ lead to larger estimation error