Example: Least-squares navigation

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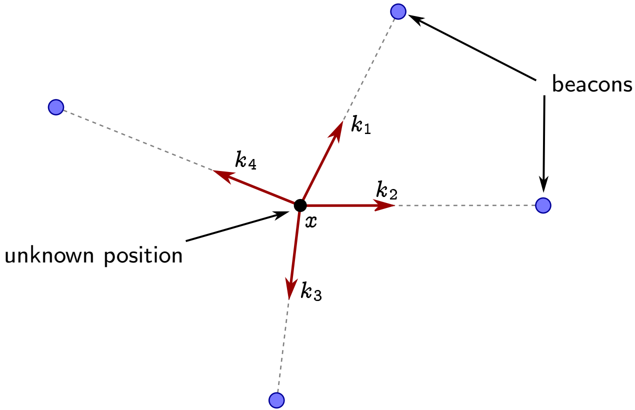
Navigation from range measurements

navigation using range measurements from *distant* beacons



beacons far from unknown position $x \in \mathbb{R}^2$, so linearization around $x = 0$ (say) nearly exact

Navigation

ranges $y \in \mathbb{R}^4$ measured, with measurement noise v:

$$y = \begin{bmatrix} k_1^T \\ k_2^T \\ k_3^T \\ k_4^T \end{bmatrix} x + v$$

where k_i is unit vector from 0 to beacon i

- **problem:** estimate $x \in \mathbb{R}^2$, given $y \in \mathbb{R}^4$
- measurement errors are independent, Gaussian, with standard deviation 2 (details not important, roughly a $2\!:\!1$ measurement redundancy ratio)
- actual position is $x = (5.59, 10.58)$;
- measurement is $y = (-11.95, -2.84, -9.81, 2.81)$

Just enough measurements method

y_1 and y_2 suffice to find x (when $v = 0$)

compute estimate $\hat{x}$ by inverting top (2×2) half of A:

$$\hat{x} = B_{\mathrm{je}} y = \begin{bmatrix} 0 & -1.0 & 0 & 0 \\ -1.12 & 0.5 & 0 & 0 \end{bmatrix} y = \begin{bmatrix} 2.84 \\ 11.9 \end{bmatrix}$$

(norm of error: 3.07)

Least-squares method

compute estimate $\hat{x}$ by least-squares:

$$\hat{x} = A^{\dagger} y = \begin{bmatrix} -0.23 & -0.48 & 0.04 & 0.44 \\ -0.47 & -0.02 & -0.51 & -0.18 \end{bmatrix} y = \begin{bmatrix} 4.95 \\ 10.26 \end{bmatrix}$$

(norm of error: 0.72)

- B_{je} and $A^{\dagger}$ are both left inverses of A
- larger entries in B lead to larger estimation error