Example: Input Design

Examples (ideas only, no details)

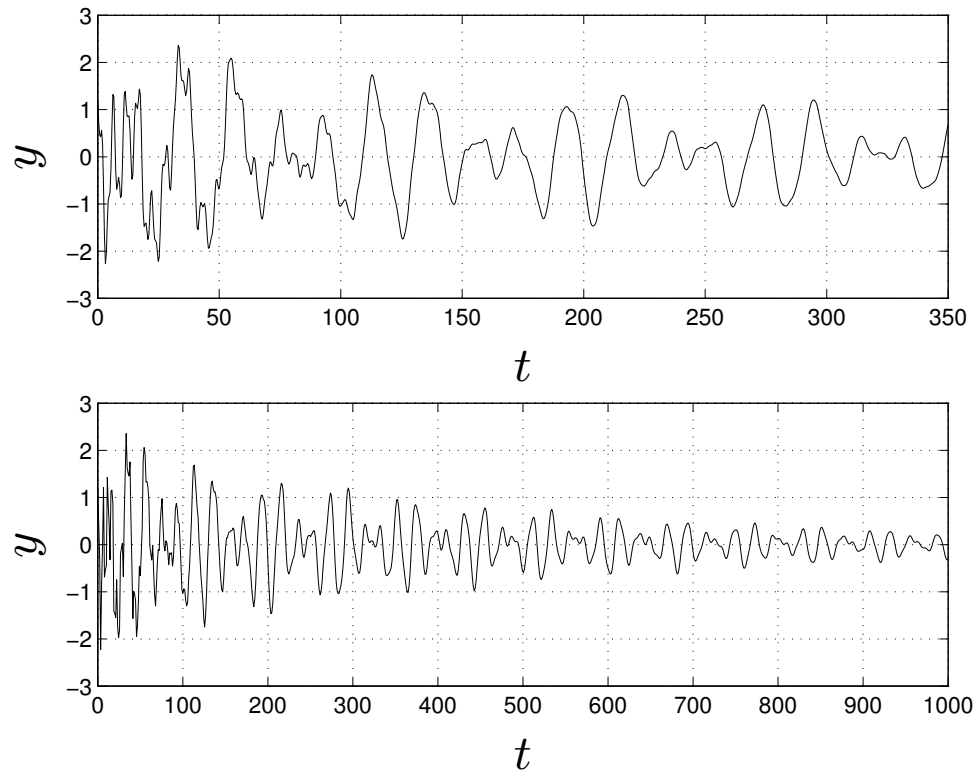
- let's consider a specific system

$$\dot{x} = Ax, \qquad y = Cx$$

  with $x(t) \in \mathbb{R}^{16}$, $y(t) \in \mathbb{R}$ (a '16-state single-output system')

- model of a lightly damped mechanical system, but it doesn't matter

Typical output

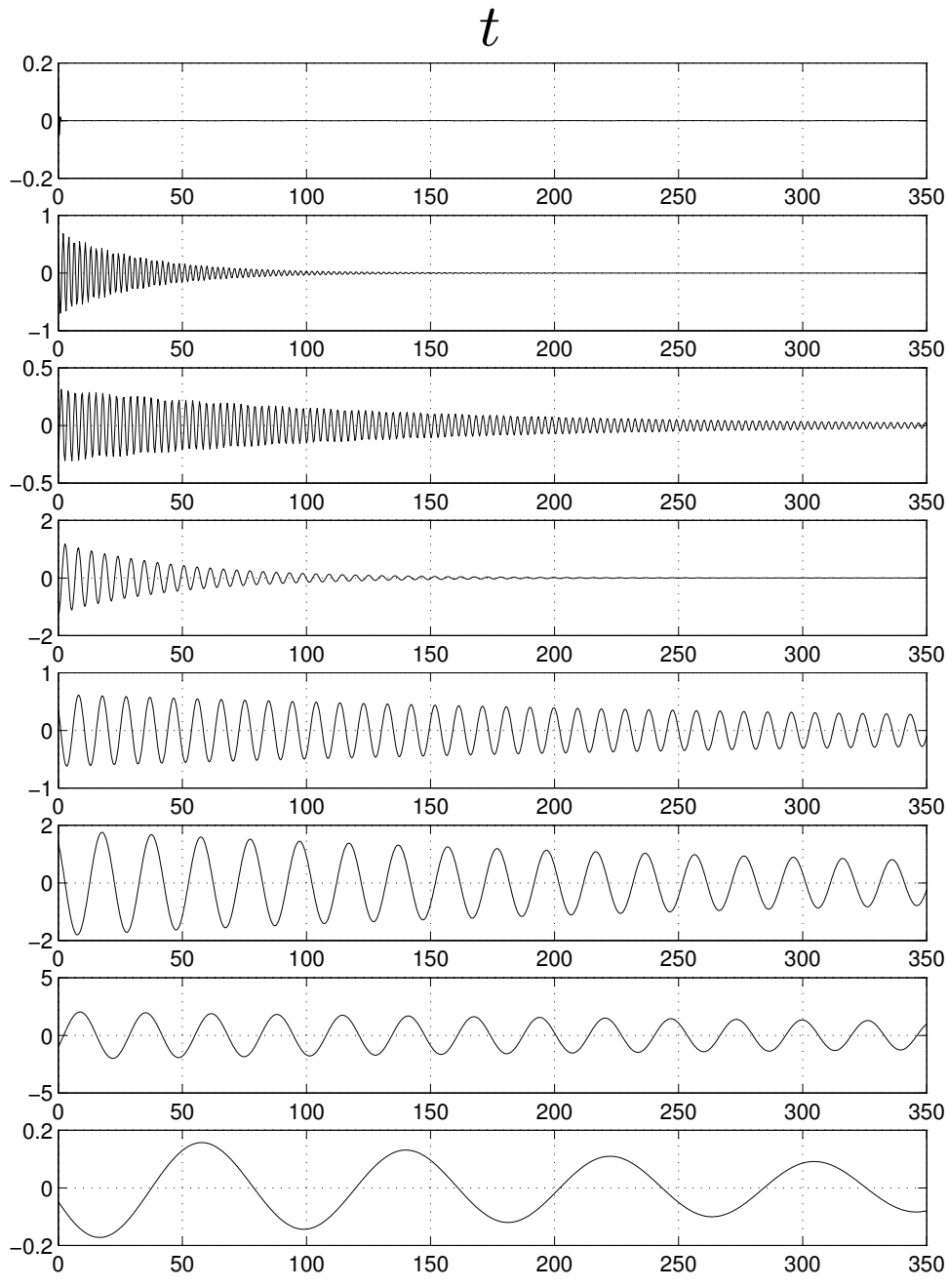


- output waveform is very complicated; looks almost random and unpredictable

Modal components

we'll see that such a solution can be decomposed into much simpler (modal) components

(idea probably familiar from 'poles')



Input design

add two inputs, two outputs to system:

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad x(0) = 0$$

where $B \in \mathbb{R}^{16\times 2}$, $C \in \mathbb{R}^{2\times 16}$ (same A as before)

problem: find appropriate $u : \mathbb{R}_+ \to \mathbb{R}^2$ so that $y(t) \to y_{\rm des} = (1, -2)$

simple approach: consider static conditions (u, x, y constant):

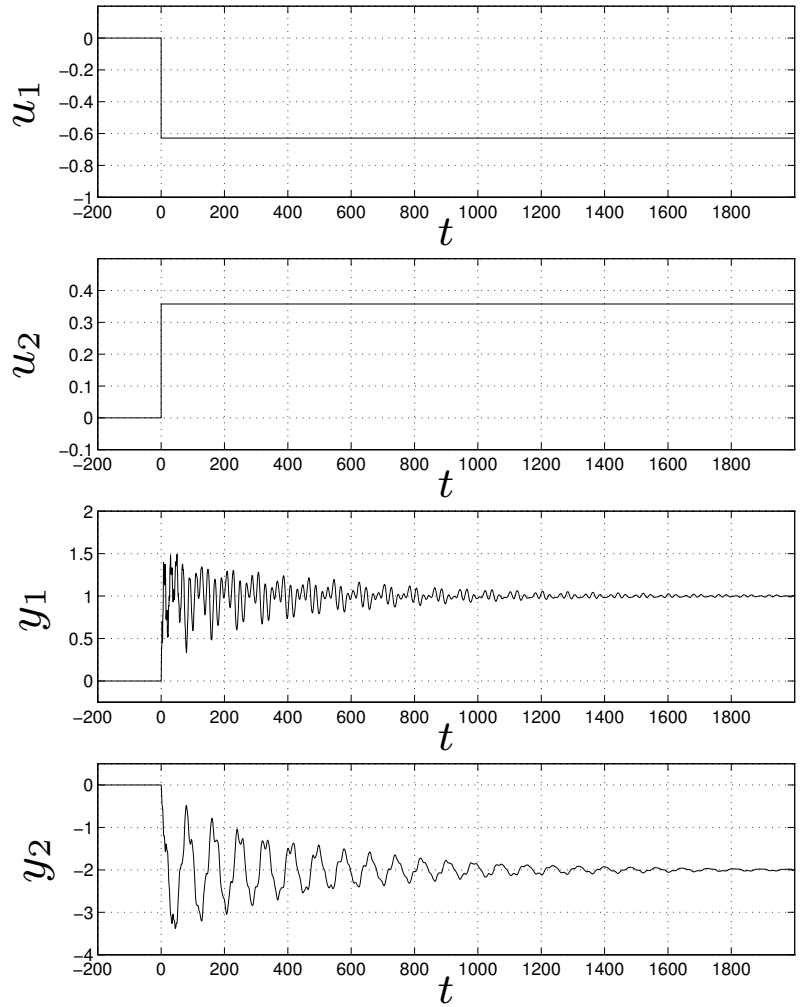
$$\dot{x} = 0 = Ax + Bu_{\rm static}, \quad y = y_{\rm des} = Cx$$

solve for u to get:

$$u_{\rm static} = \left(-CA^{-1}B\right)^{-1} y_{\rm des} = \begin{bmatrix} -0.63 \\ 0.36 \end{bmatrix}$$

Simple approach

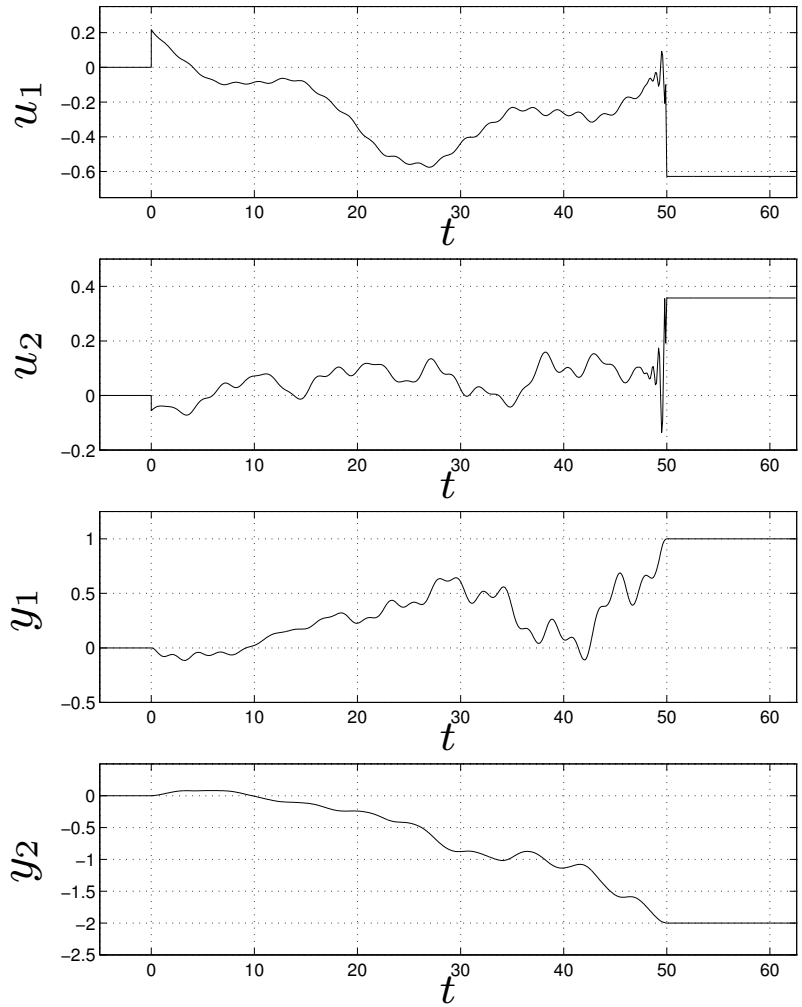
let's apply $u = u_{\text{static}}$ and just wait for things to settle:



. . . takes about 1500 sec for $y(t)$ to converge to y_{des}

Faster convergence

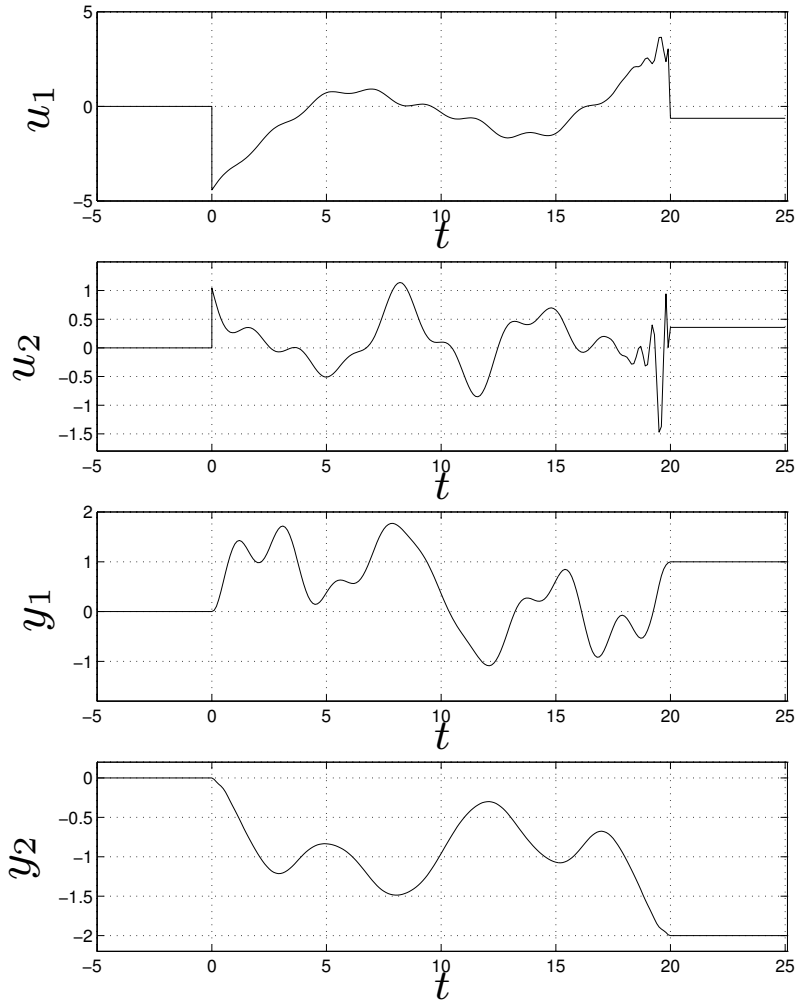
using very clever input waveforms (EE263) we can do much better, *e.g.*



. . . here y converges *exactly* in 50 sec

Still faster

in fact by using larger inputs we do still better, *e.g.*



. . . here we have (exact) convergence in 20 sec

in this course we'll study

- how to synthesize or design such inputs
- the tradeoff between size of u and convergence time