Lecture 1
Matrix Terminology and Notation

- matrix dimensions
- column and row vectors
- special matrices and vectors

Matrix dimensions

a *matrix* is a rectangular array of numbers between brackets

examples:

$$A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & -3 \\ 12 & 0 \end{bmatrix}$$

dimension (or size) always given as (numbers of) rows $\times$ columns

- A is a 3×4 matrix, B is 2×2
- the matrix A has four columns; B has two rows

$m \times n$ matrix is called *square* if $m = n$, *fat* if $m < n$, *skinny* if $m > n$

Matrix coefficients

coefficients (or entries) of a matrix are the values in the array

coefficients are referred to using double subscripts for row, column

A_{ij} is the value in the ith row, j column of A; also called i, j entry of A

i is the *row index* of A_{ij}; j is the *column index* of A_{ij}

(here, A is a matrix; A_{ij} is a number)

example: for $A = \begin{bmatrix} 0 & 1 & -2.3 & 0.1 \\ 1.3 & 4 & -0.1 & 0 \\ 4.1 & -1 & 0 & 1.7 \end{bmatrix}$, we have:

$A_{23} = -0.1$, $A_{22} = 4$, but A_{41} is meaningless

the row index of the entry with value -2.3 is 1; its column index is 3

Column and row vectors

a matrix with one column, $i.e.$, size $n \times 1$, is called a (column) *vector*

a matrix with one row, $i.e.$, size $1 \times n$, is called a *row vector*

'vector' alone usually refers to column vector

we give only one index for column & row vectors and call entries *components*

$$v = \begin{bmatrix} 1 \\ -2 \\ 3.3 \\ 0.3 \end{bmatrix} \qquad w = \begin{bmatrix} -2.1 & -3 & 0 \end{bmatrix}$$

- v is a 4-vector (or 4×1 matrix); its third component is $v_3 = 3.3$
- w is a row vector (or 1×3 matrix); its third component is $w_3 = 0$

Matrix equality

$A = B$ means:

- A and B have the same size
- the corresponding entries are equal

for example,

- $\begin{bmatrix} -2 \\ 3.3 \end{bmatrix} \neq \begin{bmatrix} -2 & -3.3 \end{bmatrix}$ since the dimensions don't agree
- $\begin{bmatrix} -2 \\ 3.3 \end{bmatrix} \neq \begin{bmatrix} -2 \\ 3.1 \end{bmatrix}$ since the 2nd components don't agree

Zero and identity matrices

$0_{m\times n}$ denotes the $m \times n$ **zero matrix**, with all entries zero

I_n denotes the $n \times n$ **identity matrix**, with

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$0_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$0_{n\times 1}$ called *zero vector*; $0_{1\times n}$ called *zero row vector*

convention: usually the subscripts are dropped, so you have to figure out the size of 0 or I from context

Unit vectors

e_i denotes the ith **unit vector**: its ith component is one, all others zero

the three unit 3-vectors are:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

as usual, you have to figure the size out from context

unit vectors are the columns of the identity matrix I

some authors use $\mathbf{1}$ (or e) to denote a vector with all entries one, sometimes called the **ones vector**

the ones vector of dimension 2 is $\mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$