Lecture 1
Matrix Terminology and Notation

• matrix dimensions
• column and row vectors
• special matrices and vectors
Matrix dimensions

A matrix is a rectangular array of numbers between brackets.

Examples:

\[
A = \begin{bmatrix}
0 & 1 & -2.3 & 0.1 \\
1.3 & 4 & -0.1 & 0 \\
4.1 & -1 & 0 & 1.7
\end{bmatrix}, 
B = \begin{bmatrix}
3 & -3 \\
12 & 0
\end{bmatrix}
\]

dimension (or size) always given as (numbers of) rows \(\times\) columns

- \(A\) is a \(3 \times 4\) matrix, \(B\) is \(2 \times 2\)
- the matrix \(A\) has four columns; \(B\) has two rows

\(m \times n\) matrix is called square if \(m = n\), fat if \(m < n\), skinny if \(m > n\)
Matrix coefficients

**coefficients** (or entries) of a matrix are the values in the array

coefficients are referred to using double subscripts for row, column

\( A_{ij} \) is the value in the \( i \)th row, \( j \) column of \( A \); also called \( i, j \) entry of \( A \)

\( i \) is the **row index** of \( A_{ij} \); \( j \) is the **column index** of \( A_{ij} \)

(here, \( A \) is a matrix; \( A_{ij} \) is a number)

**example:** for \( A = \begin{bmatrix}
0 & 1 & -2.3 & 0.1 \\
1.3 & 4 & -0.1 & 0 \\
4.1 & -1 & 0 & 1.7
\end{bmatrix} \), we have:

\( A_{23} = -0.1 \), \( A_{22} = 4 \), but \( A_{41} \) is meaningless

the row index of the entry with value \(-2.3\) is 1; its column index is 3
Column and row vectors

a matrix with one column, \( i.e., \) size \( n \times 1 \), is called a (column) vector

a matrix with one row, \( i.e., \) size \( 1 \times n \), is called a row vector

‘vector’ alone usually refers to column vector

we give only one index for column & row vectors and call entries components

\[
v = \begin{bmatrix}
1 \\
-2 \\
3.3 \\
0.3
\end{bmatrix} \quad w = \begin{bmatrix}
-2.1 \\
-3 \\
0
\end{bmatrix}
\]

• \( v \) is a 4-vector (or \( 4 \times 1 \) matrix); its third component is \( v_3 = 3.3 \)
• \( w \) is a row vector (or \( 1 \times 3 \) matrix); its third component is \( w_3 = 0 \)
Matrix equality

\( A = B \) means:

- \( A \) and \( B \) have the same size
- the corresponding entries are equal

for example,

- \[
\begin{bmatrix}
-2 \\
 3.3 \\
\end{bmatrix}
\neq
\begin{bmatrix}
-2 & -3.3 \\
\end{bmatrix}
\] since the dimensions don’t agree
- \[
\begin{bmatrix}
-2 \\
 3.3 \\
\end{bmatrix}
\neq
\begin{bmatrix}
-2 \\
 3.1 \\
\end{bmatrix}
\] since the 2nd components don’t agree
Zero and identity matrices

$0_{m \times n}$ denotes the $m \times n$ zero matrix, with all entries zero.

$I_n$ denotes the $n \times n$ identity matrix, with

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$0_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$0_{n \times 1}$ called zero vector; $0_{1 \times n}$ called zero row vector.

**Convention:** usually the subscripts are dropped, so you have to figure out the size of $0$ or $I$ from context.
Unit vectors

e_i denotes the \( i \)-th unit vector: its \( i \)-th component is one, all others zero

the three unit 3-vectors are:

\[
e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

as usual, you have to figure the size out from context

unit vectors are the columns of the identity matrix \( I \)

some authors use \( \mathbf{1} \) (or \( e \)) to denote a vector with all entries one, sometimes called the ones vector

the ones vector of dimension 2 is \( \mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)